OUTPUT TRACKING OF SOME CLASS NON-MINIMUM PHASE NONLINEAR SYSTEMS

by Firman Et Al

OUTPUT TRACKING OF SOME CLASS NON-MINIMUM PHASE NONLINEAR SYSTEMS

FIRMAN[1], JANSON NAIBORHU[2], ROBERD SARAGIH [3]

[1,2,3]Industrial & Financial Mathematics Group, ITB,
fina@students.itb.ac.id, janson@math.itb.ac.id, roberd@math.itb.ac.id

Abstract. In this paper we will design an input control to track the output of a non-minimum phase nonlinear system. The design of the input control is based on an input-output linearization method and gradient descent control. To perform the design of the input control, the other output should be selected such that the systems becomes minimum phase systems. Furthermore, the desired output of the output which has been selected will be set based on the desired output of the original system.

Key words and Phrases: input-output linearization, steepest descent control, minimum phase system,non-minimum phase system.

1. Introduction

A system called minimum phase systems if the origin of the zero dynamics is asymptotically stable, but if zero dynamics is unstable, then the system called non-minimum phase [6] . In [1], Isidori has shown that for the minimum phase systems, then it can be choose a static control law such that the output of the system goes to zero while keeping the state of the system bounded locally. While that in [2] has introduced a dynamic feedback control which is a modification of the steepest descent control. The modified steepest descent control success to handle the problem that arise if relative degree of the nonlinear system not well defined. Recently, output tracking problems on nonlinear non-minimum phase systems have been investigated intensively. The stable inversion proposed in [3], [4] is an iterative solution to the tracking problem with the unstable zero dynamics. This method requires the system to have well defined relative degree and hyperbolic dynamics, i.e. no eigenvalues on imaginary axis. In [5], proposed the control design procedure for the output tracking. The design procedure consists of two steps. In the first step, the standard input output linearization is applied. In the second step, we group a subset of the output with the internal dynamics as one subsystems, which is usually nonlinear, and the rest of output as the other subsystem which is linear, the nonlinear subsystems is linearized about its equilibrium. In [7], Riccardo Marino and Patrizio Tomei have shown how to design a globally stabilizing dynamic output feedback controller of order $n + 2(\rho - 1)$(n is the system order, ρ is the relative degree) for a class of nonlinear nonminimum phase systems. The

system are required to be minimum phase with respect to a linear combination of the state variables. In [8], the asymptotic output tracking which is a class of causal nonminimum phase uncertain nonlinear systems is achieved by using higher order sliding modes (HOSM) without reduction of the input-output dynamics order. In [9], a new nonlinear dynamic controller is described based on the gradient descent control. Performance index is generated by error of output system from output desired value and internal state of the system. adding of an internal state to maintain the stability of internal dynamic of the system.
In this paper, we will design the input control which ensures that the nonlinear system is stable asymptotically. If the system has relative degree well defined, we used the input output linearization method [1] to design input control. Then if the relative degree of the system is not well defined, to design of the input control based on the modification of the steepest descent control. Modification is the addition of an input artivisial of the steepest descent control. Furthermore, the desired output of the output which has been selected will be set based on the desired output of the original system.

2. Output Tracking

We will investigate the output tracking for a non-minimum phase nonlinear system. The non-minimum phase system in the following form :

$$\dot{x} = \mathbf{A}x + bu + \varphi(y), x \quad \mathbf{R}^n, u \quad \mathbf{R} \qquad (1)$$

$$y = x_1 \qquad (2)$$

in which $\varphi(y)$ is a smooth vector field in $\mathbf{R}^n$ with $\varphi(0) = 0$, $b = [0,\ldots,0,b_r,\ldots,b_n]^T$ with $b_r 6= 0$,

$$\mathbf{A} = \begin{pmatrix} 0 & 1 & \ldots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \ldots & 1 \\ 0 & 0 & \ldots & 0 \end{pmatrix}.$$

Our objective is to make the output system (1)-(2) tracks the desired output while keeping the state bounded. To keep the state bounded is difficult for non-minimum phase system. In this paper, we design a controller such that the output system (1)-(2) tracks the desired output while keeping the state bounded. To perform the design of the input control, the other output should be selected such that the system (1) becomes minimum phase with respect to a new output. Thus, in this paper we assume that

Assumption 1. *There exists a linear combination of the state variables* $\mu = t_1x$ *such that the zero-dynamics (1)-(2) is asymptotically stable.*

We consider the system (1)-(2). Based on assumption 1, consider now a new output

$$z_1 = \mu = t_1x = (t_{11},\ldots,t_{1n})x$$

with the relative degree of system (1) with respect to μ is still equal to r. The system (1) can be transformed to

$$\dot{z}_1 = z_2 \quad \dot{z}_2 = z_3$$

$$\vdots \qquad (3)$$

$$\dot{z}_r = b(z) + a(z)u, \quad \dot{\eta} = q(z) \quad y = z_1.$$

Before applying the control law, we have to set up the output desired for z_1, i.e. z_{1d}. In this paper we consider the system which satisfies the following assumption

Assumption 2. *$\lambda_l(x) = x_l$, $l \in \{2,...,n\}$ then $\dot{x}_l = f_l(x_l, x_1)$ can be solved by substituting $x_1 = y_d(t)$*

Thus, $\lambda_{ld} = x_l(t)$.
Based on assumption 2, we obtain

$z1d = t11yd + t12\lambda 2d + \cdots + t1n\lambda nd$

let $e_1 = z_1 - z_{1d}$, $e_2 = \dot{z}_1 - \dot{z}_{1d}$, $\cdots$, $e_r = z_1^{(r-1)} - z_{1d}^{(r-1)}$. Thus

$$\dot{e}_k = e_{k+1}, \ 1,...,r-1$$

$$\dot{e}_r = b(e + z_d, \eta) + a(e + z_d, \eta)u - z_{1d}^{(r)}, \qquad (4)$$

$$\dot{\eta} = q(e + z_d, \eta, \eta).$$

By choosing

$$u = \frac{1}{a(e + z_d, \eta)}\left(-b(e + z_d, \eta) + z_{1d}^{(r)} - \sum_{i=1}^{r} c_{(i-1)}(e_1^{(i-1)})\right) \qquad (5)$$

where $c_0, c_1, ..., c_{r-1}$ are real numbers,
$e(t) = col(e_1(t).\dots, e_r(t))$ *and* $z_d(t) = col(z_{1d}(t), z_{1d}^{(1)}(t), \dots, z_{1d}^{(r-1)}(t))$

Then system (4) can be written as

$$\dot{e} = \mathbf{A}e, \qquad (6)$$

$$\dot{\eta} = q(e + z_d, \eta), \qquad (7)$$

where the matriks **A** has a characteristic polynomial : $p(s) = c_0 + c_1 s + \cdots + cr{-}1sr{-}1 + sr$.

By choosing the value of $c_i; i = 0,...,r$ such that all the roots oh the polynomial $p(s)$ have negative real part, and applying the control law (5), then the equilibrium point $(e,\eta) = (0,0)$ of the system (4) is asymptotically stable. (see Proposition 4.5.1 in [1]).
Thus e_1 tend to zero if time t goes to infinity. Then z_1 tend to z_{1d} if time t goes to infinity. Thus x_1 tracks to the desired output $y_d(t)$.

Next, if the relative degree of the system (1)-(2) is not well defined.
We construct the performance index as a descent function as follow :

$$F(z_1, \dot{z}_1, \dots, z_1^{(r)}(t)) = \left(\sum_{j=0}^{r} a_j (z_1 - z_{1d})^{(j)}\right)^2 . \qquad (8)$$

By"Trajectory Following Method" [10], the control u is determined from the differential equation

$$\dot{u} = -\frac{\partial F}{\partial u} = -2a_r \left(\sum_{j=0}^{r} a_j (z_1 - z_{1d})^{(j)} \right) \frac{\partial (z_1 - z_{1d})^{(r)}}{\partial u}, \qquad (9)$$

where the control law in (9) is called the steepest descent control [2]. Furthermore calculate the time derivative of descent function (8) along the trajectory of the extended system

$$\begin{aligned} \dot{z}_k &= z_{k+1},\ k = 1, \ldots, r-1 & (10) \\ \dot{z}_r &= a(z\ ,\ \eta)\ +\ b(z\ ,\ \eta)u & (11) \\ \dot{\eta} &= q(z\ ,\ \eta) & (12) \\ \dot{u} &= -2a_r \left(\sum_{j=0}^{r} a_j (z_1 - z_{1d})^{(j)} \right) \frac{\partial (z_1 - z_{1d})^{(r)}}{\partial u}. & (13) \end{aligned}$$

Now, we have

$$\dot{F}(y, \dot{y}, \ldots, y^{(r)}) = \left(\frac{\partial F}{\partial z} \dot{z} + \frac{\partial F}{\partial \eta} \dot{\eta} + \frac{\partial F}{\partial u} \dot{u} \right). \qquad (14)$$

From equation (14), we see that the value of time derivative of the descent function along the trajectory of the extended system can not be guaranteed to be less than zero for $t \geq 0$.

Now we modify the steepest descent control (9) by adding an artificial input v.

Then the extended system (1) becomes

$$\begin{aligned} \dot{x} &= \mathbf{A}x + bu + \varphi(y),\ x \in \mathbf{R}^n,\ u \in \mathbf{R} & (15) \\ \dot{u} &= -\frac{\partial F}{\partial u} + v. & (16) \end{aligned}$$

From equation (14), we have

$$\dot{F}(y, \dot{y}, \ldots, y^{(r)}) = \left(\frac{\partial F}{\partial x} \dot{x} + \frac{\partial F}{\partial u} \dot{u} \right) = \frac{\partial F}{\partial x} \dot{x} + \frac{\partial F}{\partial u} \left(-\frac{\partial F}{\partial u} + v \right). \qquad (17)$$

From Sontag formula, we get

$$v = \frac{1}{\frac{\partial F}{\partial u}} \left(-\frac{\partial F}{\partial x} \dot{x} - \sqrt{\left(\frac{\partial F}{\partial x} \dot{x} \right)^2 + \left(\frac{\partial F}{\partial u} \right)^2} \right). \qquad (18)$$

The control law in equation (16) is called as modified steepest descent control.

Based on the modified steepest descent control, then $\dot{F}(y, \dot{y}, \ldots, y^{(r)}) < 0$, if $\left(\sum_{j=0}^{r} a_j (z_1 - z_{1d})^{(j)} \right) \neq 0$. Thus, if we choose a_j such that the polynomial $p(s) = a_0 + a_1 s + \cdots + a_{r-1} s^{r-1} + s^r$ is Hurwitz, z_1 tend to z_{1d} if time t goes to infinity. Thus x_1 tracks to the desired output $y_d(t)$.

Example 1. Consider the nonlinear system (SISO)

$$\begin{aligned} \dot{x}_1 &= x_2\ +\ 2x_1^2 \\ \dot{x}_2 &= x_3\ -\ u + 2x_1^2 \qquad (19) \end{aligned}$$

$$\dot{x}_3 = u - 4x_1^2$$

$$y = x_1, \ y_d = sin(t).$$

6

The nonlinear system (19) has relative degree 2 at any point x_0 (relative degree of the system is well defined). In normal form, the nonlinear system (19) becomes

$$\begin{aligned} \dot{z}_1 &= z_2 \\ \dot{z}_2 &= = 4z_1^2 - z_2 + 4z_1 z_2 + \eta - u \\ \dot{\eta} &= \eta - z_2. \end{aligned} \tag{20}$$

Because the stability of zero dynamics is unstable, the nonlinear system (19) is the non-minimum phase. Now, redefining output $z_1 = \mu = x_1 + 2x_2 + 2x_3$. By considering the new output, the relative of the system (19) is 2 at any point x_0
and normal form

$$\begin{aligned} \dot{z}_1 &= z_2 \\ \dot{z}_2 &= b(z) + a(z)u \\ \dot{\eta} &= -\eta + z_2, \end{aligned} \tag{21}$$

where $b(z) = x_3 - 6x_1^2 - 4x_1x_2 - 8x_1^3$, $a(z) = 1$. The zero-dynamics of the system (19) are asymptotical stable. Thus the system (19) is the minimum phase with

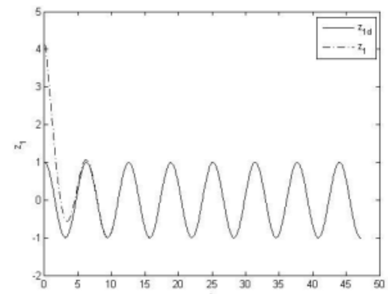

Figure 1 : Output tracking z_1 *to* z_{1d}

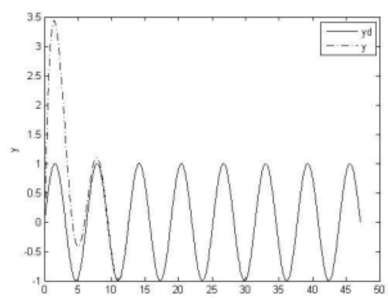

Figure 2: Output tracking (original system) y *to* y_d

respect to a new output.

Let $y_d(t) = sin(t) = x_{1d}(t)$. Next, we chose $z_{1d}(t)$ such that if $z_1(t)$ tracks $z_{1d}(t)$, then $y(t)$ tracks to the desired output $y_d(t)$. By replacing x_1 with $x_{1d}(t) = sin(t)$, then we have $x_{2d} = cos(t) - 2sin^2 t$. By replacing x_2 with $x_{2d}(t)$, we have a differential equation $\dot{x}_3 - x_3 = sin(t) + 2sin(2t) - 2sin^2 t$.

Thus $x_{3d} = -1/2cos(t) - 1/2sin(t) - 2cos^2 t + 2$. Next, $z_{1d} = x_{1d} + 2x_{2d} + 2x_{3d} = cos(t)$. According to (5), the input control is

$$u = -b(z) + \ddot{z}_{1d} - c_0(z_1 - z_{1d}) - c_1(\dot{z}_1 - \dot{z}_{1d})$$

Simulation results are shown in Figure 1 and in Figure 2 for constants: $c_0 = 6$, $c_1 = 10$. Initial value: $x_1(0) = 0$, $x_2(0) = 1$, $x_3(0) = 1$.

In Figure 1, the output which has been selected such that the system become minimum phase track the desired output z_{1d}.

In Figure 2, the output of the original system track the desired output y_d.

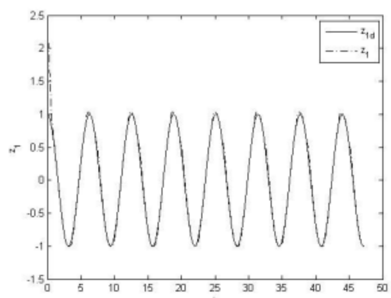

Figure 3: Output tracking z_1 *to* z_{1d}

Example 2.

$$\begin{aligned} \dot{x}_1 &= x_2 + 2x_1^2 \\ \dot{x}_2 &= x_3 - x_2 u - 2x_1^2 \qquad (22) \\ \dot{x}_3 &= x_2 u \\ y &= x_1,\ y_d = sin(t). \end{aligned}$$

The nonlinear system (22) has relative degree 2 at any point $x_0 \neq 0$ (relative degree of the system is not well defined). The system (22) is the non-minimum phase. By considering the new output $z_1 = \mu = x_1 + 2x_2 + 2x_3$, the zero dynamic are $\dot{\eta} = -\eta$. Therefore the system (22) is minimum phase with respect to the new output. By the same method as in example 1, obtained $z_{1d} = x_{1d} + 2x_{2d} + 2x_{3d} = cos(t)$.

According to (9), the modified steepest descent control is $u^{\cdot} = -2x_2 a_2(a_0(z_1 - z_{1d}) + a_1(\dot{z}_1 - \dot{z}_{1d}) + a_2(\ddot{z}_1 - \ddot{z}_{1d})) + v$, (23) with v as in (18). Simulation results are shown in Figure 3 and in Figure 4 for constants: $a_0 = 35$, $a_1 = 12$, $a_2 = 1$. Initial value: $x_1(0) = 0$, $x_2(0) = 1$, $x_3(0) = 1$, $u(0) = 1$.

In Figure 3, By modified steepest descent control, the the output which has been selected such that the system become minimum phase track the desired output z_{1d}. In Figure 4, the output of the original system track the desired output y_d.

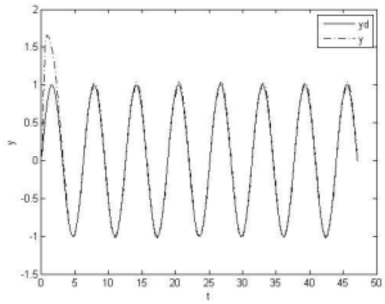

Figure 4: Output tracking (original system) *y to* y_d

3. Conclusions

In this paper, we have investigated the output tracking for a class of nonlinear non-minimum phase system (1)-(2). The input control has been designed for the output tracking. To perform the design of the input control, the system (1) are required to be minimum phase with respect to a new output, where the new output is a linear combination of the state variables. Furthermore, the desired new output will be set based on the desired output of the original. If the system (1) has relative degree well defined with respect to the new output, we used the input output linearization method to design the input control. Then if the relative degree of the system (1) is not well defined with respect to the new output , to design of the input control based on the modification of steepest descent control.

References

[1] A. Isidori.*Nonlinear Control Systems: An Introduction*. Secon Edition Springer-Verlag Berlin, Heidelberg 1989.

[2] J Naiborhu.*Trajectory Following Method on Output Regulation of Affine Nonlinear Control Systems with Relative Degree not Well Defined*. ITB J.Sci.,Vol. 43A,No.1,2011,73-84.

[3] D.Chen and B.Paden (1996), Stable inversion of nonlinear non-minimum Phase systems, *Int.J.Control* , Vol.64,No.1, 45-54.

[4] D.Chen (1993), Iterative solution to stable inversion of nonlinear non-minimum Phase systems, *Proc. American Control Conference*, June, 2960-2964.

[5] Dong Li. (2005), Output Tracking of Nonlinear Nonminimum Phase Systems : an Enginering Solution, *Proceedings of the 44th IEEE Conference on Decision and control, and the European Control Conference*, Sevilla,Spain,Dec 12-15, 3462-3467.

[6] Hassan K.Khalil. (2002), Nonlinear Systems, *Prentice Hall, New Jersey* , Third Edition.

[7] Riccardo Marino, Patrizio Tomei. (2004), A class of Globally Output Feedback Stabilizable Nonlinear Non-minimum Phase Systems, *43rd IEEE Conference on Decision and Control*, Atlantis, Paradise Island,Bahamas Des 14-17, 4909-4914.

[8] S. Baev, Y.Shtessel, I. Shkolnikov. (2007), HOSM driven outputvtracking in the nonminimum-phase causal nonlinear Systems, *Proceeding of the 46th IEEE Conference on Decision and Control*, New Orleands,LA,USA, Des 12-14, 3715-3720.

[9] J. Naiborhu.*Output Tracking of Nonlinear Non-minimum Phase Systems by Gradient Descent Control*. proceeding of the IASTED International Conference Identification, Control, and Applications (ICA 2009),August 17-19, 2009, Honolulu Hawaii, USA, 110-115

[10] Vincent,T.L. and W.J. Grantham,*Nonlinear and Optimal Control Systems*, John Wiley and Sons, Inc., New York, 1997.

OUTPUT TRACKING OF SOME CLASS NON-MINIMUM PHASE NONLINEAR SYSTEMS

ORIGINALITY REPORT

11%	10%	9%	0%
SIMILARITY INDEX	INTERNET SOURCES	PUBLICATIONS	STUDENT PAPERS

PRIMARY SOURCES

1	R. Marino, P. Tomei. "A class of globally output feedback stabilizable nonlinear non-minimum phase systems", 2004 43rd IEEE Conference on Decision and Control (CDC) (IEEE Cat. No.04CH37601), 2004 Publication	3%
2	Dimitar Ho, J. Karl Hedrick. "Control of nonlinear non-minimum phase systems with input-output linearization", 2015 American Control Conference (ACC), 2015 Publication	2%
3	css.paperplaza.net Internet Source	2%
4	resolver.caltech.edu Internet Source	2%
5	repository.lib.cuhk.edu.hk Internet Source	2%
6	Monia Charfeddine, Khalil Jouili, Naceur Benhadj Braiek. "chapter 6 Approximate	2%

Input-Output Feedback Linearization of Non-Minimum Phase System using Vanishing Perturbation Theory", IGI Global, 2015

Publication

Exclude quotes	On	Exclude matches	< 2%
Exclude bibliography	Off		